Extended TOPSIS for Group Decision Making with Linguistic Quantifiers and Concept of Majority Opinion

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***Abstract*—This paper presents a fuzzy extension of TOPSIS (technique for order performance by similarity to ideal solution) with a new quantifier guided distance metric and majority opinion aggregator for multi-criteria decision making in a group decision environment. The proposed distance metric is based on OWA aggregators and provides an opportunity to use linguistic quantifiers to have linguistic definitions for proximity. On the other hand, the majority opinion aggregator is used to make a consensual judgment for synthesizing the individual opinions. A human resource selection problem is considered as the case study, and the proposed algorithm is employed to solve that. Simulation results show that our algorithm is more advantageous in reflecting opinions of the majority of decision makers and providing more confidence for their decisions.**

***Keywords*—Multi-Criteria Decision Making, Group Decision Making, TOPSIS, OWA, Majority Opinion.**

I. Introduction

In all real decision making problems multiple criteria are considered to evaluate the choices. On the other hand, many decision making problems within organization are performed in a collaborative effort [1]. This study proposes a fuzzy approach for multi-criteria group decision making (MCGDM) under uncertainty using technique for order performance by similarity to ideal solution (TOPSIS) [2], linguistic quantifiers [3], and the concept of majority opinion [4].

TOPSIS is a popular and useful technique in dealing MCDM problems in the real words. Some of the advantages of TOPSIS according to [5] are intuitive and clear logic that represent the rationale of human choice, a scalar value that accounts for both the best and worst alternatives, ease of computation, and possibility for visualization. Linguistic quantifiers in fuzzy logic are used to generalize the concept of quantification of classical logic and helps to accomplish quantifier guided aggregations in decision making problems. In GDM, the goal is to determine a consensual judgment for each alternative that reflects the individual opinions [4]. In this respect, a majority opinion is a consensual judgment of a majority of the decision makers who have similar opinions. The concept of majority opinion is a novel idea proposed by Pasi and Yager in 2006 that can result to a great improvement for GDM techniques.

In literature, there are interesting results on group decision-making (GDM) or social choice theory and multi-criteria decision-making (MCDM) with the help of fuzzy sets theory that the interested reader is referred to [6]-[14].

The rest of this paper is organized as follows. Section II provides a definition for MCGDM, summarizes TOPSIS procedure for MCGDM problems, review OWA and IOWA operators with linguistic quantifiers, and elaborates the concept of majority opinion. In section III, our proposed algorithm is explained presenting a new distance measure based on OWA operators and modifying the concept of majority opinion to be integrated in TOPSIS for internal group aggregation. Next, the simulation results are summarized for a case study of human resource selection in section IV, and finally, the conclusions are drawn in section V.

II. Multi-Criteria Group Decision Making

In this paper, the context of multi-criteria group decision making is addressed. It is supposed that there is a group of decision makers (DMs) or experts, $DM = \{DM_1, \dots, DM_K\}$ which provide the performance value for the alternatives $A = \{A_1, \dots, A_m\}$ with respect to a set of criteria $C = \{C_1, \dots, C_n\}$, and the aim of decision making is to find the best alternative (or ranking of the alternatives) based on the overall performance in the specified criteria and different points of view. In both multi-criteria decision making (MCDM) and group decision making (GDM), there are two steps: aggregation and exploitation [6], [7], [9], [12], [15]. In MCDM, aggregation is to combine satisfaction over different criteria while GDM problem consists in combing the experts' opinions into a group collective one.

A. TOPSIS for Group Decision Making

The original TOPSIS (Technique for Order Preference by Similarity to Ideal Solution) is proposed by Hwang and Yoon in 1981 [2] is a considered as a popular algorithm for MCDM under certain conditions. The idea is to find the best alternative with the shortest distance to the positive-ideal solution and the farthest distance from the negative-ideal solution. The procedure which is presented in this section is the model proposed in [1] for group decision making.

The process for original TOPSIS (not the group model) starts with forming the decision matrix representing the satisfaction value of each criterion (attribute) with each alternative. Next, the matrix is normalized with a desired normalizing scheme, and the values are multiplied by the criteria weights. Subsequently, the positive-ideal and negative-ideal solutions are calculated, and distance of each alternative to these solutions is calculated with a distance measure. Finally, the alternatives are ranked based on their relative closeness to the ideal solution.

Now, we explain the detailed procedure for TOPSIS for group decision making described in [1].

Step 1. Making decision matrix D^k, $k = 1, \dots, K$, for each DM.

The structure for decision matrix is characterized in (1).

$$D^k = \begin{matrix} & X_1 & \cdots & X_n \\ A_1 & x_{11}^k & \cdots & x_{1n}^k \\ \vdots & \vdots & \ddots & \vdots \\ A_m & x_{m1}^k & \cdots & x_{mn}^k \end{matrix} \quad (1)$$

A_i indicates the alternative i, and X_j denotes the criterion j, and x_{ij}^k denotes the performance value for alternative A_i with respect to criterion X_j.

Step 2. Making the normalized decision matrix R^k, $k = 1, \dots, K$, for each DM.

Any normalization method can be used in this step like linear normalization, vector normalization, non-monotonic normalization, etc. in this paper, vector normalization is used which is expressed in (2).

$$r_{ij}^k = \frac{x_{ij}^k}{\sqrt{\sum_i^m (x_{ij}^k)^2}} \quad (2)$$

In original TOPSIS, the weighted normalized matrix is calculated, but in [1], the weights are manipulated in next steps. One might see the reason in [16].

Step 3. Determine the positive-ideal and negative-ideal solutions V^{k+} and V^{k-} $k = 1, \dots, K$, for each DM.

PIS and NIS are obtained as follows.

$$V^{k+} = \{r_1^{k+}, \dots, r_n^{k+}\} = \{(max_i\, r_i^k \mid j \in J), (min_i\, r_i^k \mid j \in J')\} \quad (3)$$

$$V^{k-} = \{r_1^{k-}, \dots, r_n^{k-}\} = \{(min_i\, r_i^k \mid j \in J), (max_i\, r_i^k \mid j \in J')\} \quad (4)$$

Where J is associated with the set of benefit criteria and J' is associated with the set of cost criteria.

Step 4. Calculate the separation measures from the PIS and NIS for the group.

This step can be divided in the following two steps.

Step 4a. Calculate the separation measures individually.

In this step, the separation measure from PIS and NIS are computed with a distance metric. The manipulation for Minkowski's L_p metric as the distance measure is described in (5) and (6).

$$S_i^{k+} = \left\{\sum_{j=1}^n w_j^k (v_{ij}^k - v_{ij}^{k+})^p\right\}^{1/p}, \quad \text{for } i = 1, \dots, m \quad (5)$$

$$S_i^{k-} = \left\{\sum_{j=1}^n w_j^k (v_{ij}^k - v_{ij}^{k-})^p\right\}^{1/p}, \quad \text{for } i = 1, \dots, m \quad (6)$$

where $p \geq 1$ and integer, w_j^k is the weight for the criterion j and DM k, and $\sum_{j=1}^n w_j^k = 1$, $k = 1, \dots, K$.

Note that the metric with $p = 2$ is the Euclidean distance, and the metric with $p = 1$ is the Manhattan distance.

Step 4b. Calculate the measures for the group

In this step, the measures for different DMs should be combined (aggregated) through an operation ⊗, i.e.:

$$\overline{S_i^+} = S_i^{1+} \otimes \dots \otimes S_i^{K+} \quad \text{for alternative } i \quad (7)$$

$$\overline{S_i^-} = S_i^{1-} \otimes \dots \otimes S_i^{K-} \quad \text{for alternative } i \quad (8)$$

Many operators can be employed for this purpose. In [1], geometric mean (9) and arithmetic mean (10) are used to calculate the group separation measures from PIS and NIS.

$$\overline{S_i^+} = \left(\prod_{k=1}^K S_i^{k+}\right)^{\frac{1}{K}}, \quad \overline{S_i^-} = \left(\prod_{k=1}^K S_i^{k-}\right)^{\frac{1}{K}} \quad (9)$$

$$\overline{S_i^+} = \left(\sum_{k=1}^K S_i^{k+}\right)\Big/K, \quad \overline{S_i^-} = \left(\sum_{k=1}^K S_i^{k-}\right)\Big/K \quad (10)$$

Step 4b. Calculate the relative closeness $\overline{C_i^*}$ to the ideal solution for the group.

The relative closeness is calculated according to (11) and the alternatives are ranked in descending order.

$$\overline{C_i^*} = \frac{\overline{S_i^-}}{\overline{S_i^+}} \qquad i = 1, \dots, m \quad (11)$$

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Note that $0 \leq \overline{C_i^*} \leq 1$, and larger value of $\overline{C_i^*}$ denotes the better performance of the alternative.

B. OWA and IOWA Operators

Ordered Weighted Averaging (OWA) as an aggregation operator was proposed by Yager in 1988 [17]. OWA is a mapping from $[0,1]^n \rightarrow [0,1]$ that has associated a weighting vector $W = [w_1, \dots, w_n]$ such that $w_i \in [0,1]$, and $\sum_{i=1}^{n} w_i = 1$, and is defined to aggregate a list of arguments $a_1, \dots, a_n \in [0,1]$ according to (12).

$$\mathrm{OWA}(a_1, \dots, a_n) = \sum_{i=1}^{n} b_i w_i \tag{12}$$

where b_i is the ith largets element of the a_j.

The degree of orness or optimism degree for an OWA aggregation operator denotes its closeness to OR operator and it is defined as follows.

$$orness(w_1, \dots, w_n) = \left(\frac{1}{n-1}\right) \sum_{j=1}^{n} \left((n-j).w_j\right) \tag{13}$$

Using Linguistic quantifiers is one the approaches used to determine the weights in OWA operators. Here, we assume a Regular Increasing Monotonic (RIM) linguistic quantifier $Q: [0,1] \rightarrow [0,1]$ such that $Q(0) = 0$ and $Q(1) = 1$, and consequently the OWA weighting vector can be computed from Q using (14) [17].

$$w_i = Q(i/n) - Q((i-1)/n) \tag{14}$$

Many different choices are existed for selecting the function$Q$. A popular form is $Q(p) = p^{\alpha}$ in which α is the parameter to be set. For this function, seven RIM quantifiers are suggested in [18] which are demonstrated in Table I.

TABLE I
FAMILY OF RIM QUANTIFIERS AND THEIR RELEVANT VALUES OF α AND θ

Linguistic quantifier	Parameter of quantifier (α)	Orness (θ)
At least one of them	$\alpha \rightarrow 0$	0.999
Few of them	0.1	0.909
Some of them	0.5	0.667
Half of them	1	0.500
Many of them	2	0.333
Most of them	10	0.091
All of them	$\alpha \rightarrow \infty$	0.001

Induced ordered weighted averaging (IOWA) is an extension of OWA introduced by Yager and Filev in 1999 [19]. An IOWA operator is a mapping to aggregate a list of n 2-tuples $\{\langle a_1, v_1\rangle, \dots, \langle a_n, v_n\rangle\}$ with an associated weighting vector $W = [w_1, \dots, w_n]$ such that $w_i \in [0,1]$, and $\sum_{i=1}^{n} w_i = 1$ according to the following expression.

$$\mathrm{IOWA}(\langle a_1, v_1\rangle, \dots, \langle a_n, v_n\rangle) = \sum_{i=1}^{n} w_i a_{v-\mathrm{index}(i)} \tag{15}$$

where $v - \mathrm{index}(i)$ denotes the index of the ith largest v_i.

C. Concept of Majority Opinion

As declared in [4] the concept of majority plays a key role in group decision making, i.e., in any GDM problem, “it is needed to find an overall opinion that satisfies the opinions of the majority of the DMs. [4]” OWA aggregators with the regular linguistic quantifiers is not ideal for modeling the concept of majority, e.g., OWA aggregation with the quantifier “most of” produces a value that reflex the satisfaction of the proposition “most of the criteria have to be satisfied” instead of “the satisfaction value of most of the criteria” [4]. For instance, OWA aggregation of the values (1 1 1 0.5 0 0) with the quantifier “most of” in Fig. 1 results to the value 0.35 while the majority opinion is intuitively close to 1. Indeed, what we need is an aggregation of the most similar opinions. To this end, in [4], a mechanism based on IOWA operators with an inducing ordering variable which denotes the similarity (proximity) of the elements to be aggregated is proposed to model the majority opinion.

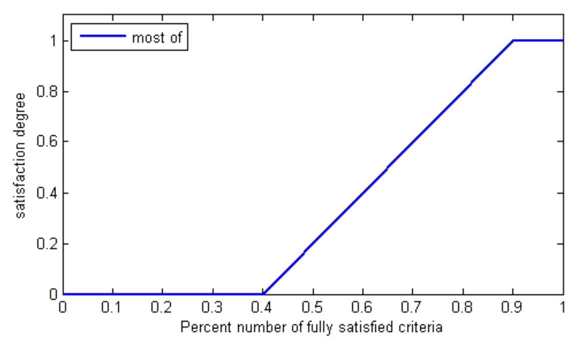

Fig. 1. A possible definition for the linguistic quantifier “most

The process described in [4] to obtain the majority opinion is as follows. Firstly, a support function $Sup(a, b)$ which represents the support from a_j for a_i is defined:

$$Sup(a_i, a_j) = \begin{cases} 1 & if\ |a_i - a_j| < \alpha \\ 0 & otherwise \end{cases} \tag{16}$$

Next, for each DM, all the supports (by other DMs) are summed to get the overall support which is expressed by s_k for DM_k, $k = 1, \dots, K$. Then the values $t_1, \dots, t_K$ are defined: $t_k = s_k + 1$. Subsequently, the elements to be aggregated are ordered in the increasing order of similarity (s_k or t_k), and the IOWA aggregator is applied to the elements with a new formula for computing the weights in (17) and the linguistic quantifier (Q) “most of” in Fig. 1.

$$w_k = \frac{Q(t_k/K)}{\sum_{i=1}^{K} Q(t_i/K)} \tag{17}$$

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where the t_k values in the above equation are in increasing order. For more detailed information, one can see [4].

III. The Proposed Algorithm

The proposed group fuzzy TOPSIS is the same as the original TOPSIS except for the distance measure and the mechanism for group aggregation. Indeed, we describe a fuzzy distance based on OWA operator and linguistic quantifiers as a new distance measure, and also we introduce a novel procedure for group aggregation based on the concept of majority opinion and IOWA operators.

A. OWA Based Distance

According to step 4a in the previous section for TOPSIS algorithm, OWA based distance is described as follows:

$$S_i^{k+} = OWA(w_j^k \left| v_{ij}^k - v_{ij}^{k+} \right|) \tag{18}$$

and

$$S_i^{k-} = OWA(w_j^k \left| v_{ij}^k - v_{ij}^{k-} \right|) \tag{19}$$

It can be shown that Manhattan distance leads to the same results as OWA-based distance with the RIM quantifier "half of" ($a = 1$).

B. Majority Opinion Aggregator

To introduce this aggregator, we use the concept of majority opinion explained in section II. However, there is a little change in definition of the support function. Indeed, a normalizing process for the opinion of DMs is used to put the degrees for different alternatives in approximately the same range, so selection of the same α for all the alternatives makes sense, and also setting the appropriate value for α becomes easier.

Hence, the support function for two DMs k and l is defined as follows:

$$Sup\left(S_i^k, S_i^l\right) = \begin{cases} 1 & if \ \frac{\left|S_i^k - S_i^l\right|}{\sum_{k=1}^{K} S_i^k} < \alpha \\ 0 & otherwise \end{cases} \tag{20}$$

Then like the procedure in majority opinion model (see section II, part C), the DMs opinions are aggregated with the IOWA operator in (21) and (22).

$$\overline{S_i^+} = IOWA(\langle S_i^{1+} | t_i^{1+} \rangle, \dots, \langle S_i^{k+} | t_i^{k+} \rangle) \tag{21}$$

$$\overline{S_i^-} = IOWA(\langle S_i^{1-} | t_i^{1-} \rangle, \dots, \langle S_i^{k-} | t_i^{k-} \rangle) \tag{22}$$

Note that IOWA operator is calculated with the linguistic quantifier described in (17).

IV. Simulation Studies

The case study which is considered in this section is a human resource selection problem for a local chemical company described in [1]. There are 17 candidates (alternatives) and 4 decision makers, and the candidates' qualification is measured through a number of objective and subjective tests. The basic data for this experiment is demonstrated in Table II and Table III. Moreover, the weights associated to DMs and for each attribute are shown in Table IV. This problem is studied in [1] using group TOPSIS algorithm with Geometric and Arithmetical mean in an internal aggregation scheme. In this section we apply our proposed algorithm to solve this problem and we compare the results with those in [1].

TABLE II
Decision Matrix of Human Resource Selection Problem — Objective Attributes [1]

No.	Objective attributes				
	Knowledge tests			Skill tests	
	Language test	Professional test	Safety rule test	Professional skills	Computer skills
1	80	70	87	77	76
2	85	65	76	80	75
3	78	90	72	80	85
4	75	84	69	85	65
5	84	67	60	75	85
6	85	78	82	81	79
7	77	83	74	70	71
8	78	82	72	80	78
9	85	90	80	88	90
10	89	75	79	67	77
11	65	55	68	62	70
12	70	64	65	65	60
13	95	80	70	75	70
14	70	80	79	80	85
15	60	78	87	70	66
16	92	85	88	90	85
17	86	87	80	70	72

A. Distance Measures Comparison

In this part, the proposed OWA based distance is compared with Euclidean and Manhattan distance. The quantifier "most of" in Table I is used for the OWA operator, and geometric mean is employed for group aggregation. The results are provided in Table V. The difference between relative closeness of two consecutive alternatives (in rank) can be used to evaluate the confidence of each algorithm, i.e., when the calculated relative closeness for two alternatives are very close to each other, the DMs might get perplexed for choosing the most satisfying one. For this purpose, we introduce three measures: 1) sum of the absolute difference between relative closeness of the consecutive alternatives, 2) minimum of the absolute difference between relative closeness of the consecutive alternatives, and 3) the range of calculated relative closeness for the alternatives. These measures for each of the three abovementioned distance measures are calculated and listed in Table VI. It can be observed the proposed OWA based distance offers more confidence in this case study. For example, the difference between relative closeness of alternative 4 and alternative 7 is 0.0003 and 0.0011 for Euclidean and Manhattan distance respectively, however, this value for OWA based measure is 0.025.

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TABLE III
DECISION MATRIX OF HUMAN RESOURCE SELECTION PROBLEM — SUBJECTIVE ATTRIBUTES [1]

No.	Subjective attributes							
	DM #1		DM #2		DM #3		DM #4	
	Panel interview	1-on-1 interview	Panel interview	1-on-1 interview	Panel interview	1-on-1 interview	Panel interview	1-on-1 interview
1	80	75	85	80	75	70	90	85
2	65	75	60	70	70	77	60	70
3	90	85	80	85	80	90	90	95
4	65	70	55	60	68	72	62	72
5	75	80	75	80	50	55	70	75
6	80	80	75	85	77	82	75	75
7	65	70	70	60	65	72	67	75
8	70	60	75	65	75	67	82	75
9	80	85	95	85	90	85	90	85
10	70	75	75	80	68	78	65	92
11	50	60	62	65	60	65	65	70
12	60	65	65	75	50	60	45	50
13	75	75	80	80	65	75	70	75
14	80	70	75	72	80	70	75	75
15	70	65	75	70	65	70	60	65
16	90	95	92	90	85	80	88	90
17	80	85	70	75	75	80	70	75

TABLE IV
WEIGHTS ON ATTRIBUTES OF HUMAN RESOURCE SELECTION PROBLEM [1]

No.	Attributes	The weights of group			
		DM #1	DM #2	DM #3	DM #4
	Knowledge test				
1	Language test	0.066	0.042	0.060	0.047
2	Professional test	0.196	0.112	0.134	0.109
3	Safety rule test	0.066	0.082	0.051	0.037
	Skill tests				
4	Professional skills	0.130	0.176	0.167	0.133
5	Computer skills	0.130	0.118	0.100	0.081
	Interviews				
6	Panel interview	0.216	0.215	0.203	0.267
7	1-on-1 interview	0.196	0.255	0.285	0.326

TABLE V
COMPARISON BETWEEN DIFFERENT DISTANCE MEASURES

No.	Manhattan distance		Euclidean distance		OWA based distance	
	C_i^*	Rank	C_i^*	Rank	C_i^*	Rank
1	0.6369	5	0.6271	5	0.5954	5
2	0.4462	11	0.4404	14	0.4407	14
3	0.8361	3	0.7860	3	0.7852	3
4	0.4393	13	0.4526	12	0.4522	13
5	0.4445	12	0.4659	11	0.5322	9
6	0.6697	4	0.6611	4	0.6591	4
7	0.4383	14	0.4523	13	0.4550	12
8	0.5683	8	0.5700	8	0.5012	11
9	0.9131	2	0.8797	2	0.8963	2
10	0.5075	10	0.5080	10	0.5062	10
11	0.1636	16	0.2096	16	0.1707	16
12	0.1322	17	0.1677	17	0.1426	17
13	0.5578	9	0.5568	9	0.5562	8
14	0.6033	6	0.5924	6	0.5936	6
15	0.3941	15	0.4091	15	0.4077	15
16	0.9200	1	0.8959	1	0.9113	1
17	0.5975	7	0.5920	7	0.5907	7

Note: Group aggregator is geometric mean for all the cases

B. *Aggregation Comparison*

In this part, we compare our majority aggregation scheme with geometric mean and arithmetic mean. Table VII shows the results for the case of Euclidean distance and different group decision making aggregators. For the majority aggregator, α is set to 0.1, and the "most of" quantifier in Fig. 1 is employed. Comparing geometric/arithmetical mean and majority aggregator, two significant changes in the order of the alternatives are detected. Rank of candidate 5 is changed from 11 to 9, and rank of candidate 8 is changed from 8 to 11. The reason for these changes can be found in Table III, taking more detailed investigation in data for candidate 5 and 8. It can be observed that the score of DM3 for alternative 5 in the interviews is much lower than other DMs, hence in the majority opinion procedure, the effect of his opinion becomes faint. On the other hand, for candidate 8, there is a problem with DM4 whose given score for the interviews is much more than other DMs.

TABLE VI
CONFIDENCE MEASURES FOR DIFFERENT DISTANCE MEASURES

Measure no.	Manhattan distance	Euclidean distance	OWA based distance
1	0.7879	0.7282	0.8291
2	0.0011	0.0003	0.0013
3	[0.1322 0.9201]	[0.1678 0.8960]	[0.0997 0.9288]

Note: Group aggregator is geometric mean for all the cases

TABLE VII
COMPARISON BETWEEN DIFFERENT GROUP AGGREGATORS

No.	Arithmetical mean		Geometric mean		Majority Opinion	
	C_i^*	Rank	C_i^*	Rank	C_i^*	Rank
1	0.6295	5	0.6271	5	0.5954	5
2	0.4407	14	0.4404	14	0.4407	14
3	0.7583	3	0.7860	3	0.7852	3
4	0.4523	13	0.4526	12	0.4522	13
5	0.4651	11	0.4659	11	0.5322	9
6	0.6591	4	0.6611	4	0.6591	4
7	0.4551	12	0.4523	13	0.4550	12
8	0.5692	8	0.5700	8	0.5012	11
9	0.8749	2	0.8797	2	0.8963	2
10	0.5063	10	0.5080	10	0.5062	10
11	0.2302	16	0.2096	16	0.1707	16
12	0.1745	17	0.1677	17	0.1426	17
13	0.5562	9	0.5568	9	0.5562	8
14	0.5936	6	0.5924	6	0.5936	6
15	0.4077	15	0.4091	15	0.4077	15
16	0.8905	1	0.8959	1	0.9113	1
17	0.5908	7	0.5920	7	0.5907	7

Note: Distance measure is Euclidean distance for all the cases

C. OWA based distance and Majority Aggregation

In this part, the whole procedure for the proposed group fuzzy TOPSIS with OWA based distance and majority opinion aggregation is applied to the human resource selection problem. For OWA operator in distance measure, the "most of" quantifier in Table I is used, while for the majority aggregator the "most of" quantifier in Fig. 1 is exploited, and α is set to 0.2. The results are listed in Table VIII. Also, the confidence measures described in part A are calculated for this experiment and compared with those for experiments of part A and B in Table IX. It can be observed that our proposed algorithm leads to more confidence.

TABLE VIII
THE RELATIVE CLOSENESS BY GROUP FUZZY TOPSIS

No.	$\overline{S_i^+}$	$\overline{S_i^+}$	$\overline{C_i^*}$	Rank
1	0.0070	0.0136	0.6592	5
2	0.0116	0.0090	0.4378	12
3	0.0030	0.0176	0.8535	3
4	0.0126	0.0096	0.4320	13
5	0.0090	0.0117	0.5639	8
6	0.0069	0.0136	0.6629	4
7	0.0123	0.0083	0.4037	14
8	0.0094	0.0083	0.4673	11
9	0.0013	0.0187	0.9318	2
10	0.0104	0.0101	0.4923	10
11	0.0159	0.0030	0.1624	16
12	0.0179	0.0018	0.0919	17
13	0.0097	0.0109	0.5296	9
14	0.0084	0.0121	0.5903	6
15	0.0129	0.0076	0.3707	15
16	0.0009	0.0188	0.9530	1
17	0.0089	0.0117	0.5676	7

Note: Group aggregator is majority opinion and distance measure is OWA based distance

V. CONCLUSION

In this paper, an algorithm for MCDM in group decision environment is presented. The main framework for the proposed method is TOPSIS which is a popular algorithm for MCDM. Indeed, the original TOPSIS is modified for GDM with an OWA based quantifier guided distance metric and a majority opinion group aggregator. A human resource selection problem is studied with the proposed algorithm, and the results of decision making are provided. The results show that the proposed algorithm outperforms the original group TOPSIS in both reflecting the concept of majority to have a better consensual judgment for the individual opinions and providing more confidence for the final decision.

TABLE IX
CONFIDENCE MEASURES FOR DIFFERENT DISTANCES AND GROUP AGGREGATORS

Measure no.	Euclidean Distance		OWA based Distance	
	Geometric mean	Majority opinion	Geometric mean	Majority opinion
1	0.7282	0.7687	0.8291	0.8611
2	0.0003	0.0018	0.0013	0.0037
3	[0.168 0.896]	[0.143 0.911]	[0.10 .929]	[0.092 0.953]

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