Conceptual Imitation Learning Based on Functional Effects of Action

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***Abstract*—In General, Learning new skills by imitation is faster, safer, and more efficient. In robotics research, imitation also provides an implicit and user-friendly mechanism for robot programming. But, according to the research in neuroscience and cognitive science, true imitation is accompanied by abstraction and conceptualization. This paper presents a method for autonomous acquisition, generalization, recognition, and regeneration of abstract (relational) concepts through perception of spatio-temporal demonstrations and identification of their functional effects. In fact, the effects help to classify the concepts based on their functional properties. As a result, the concepts are represented by prototypes which abstract different perceptual variants of a concept but make similar functional effects. Performance of the proposed algorithm is evaluated in a task of imitating a bunch of behaviors based on their emotional effects. Results of the experiments on a humanoid robot show that our model is successful for extraction, abstract representation, accurate recognition, and reproduction of the learned concepts.**

I. Introduction

Imitation plays a crucial role in every-day life of human and some animals. In fact, imitation is one of the main methods of social learning. In this regard, imitation learning is more efficient in time, energy, and safety than individual learning. There are also other types of social learning which are somehow similar to imitation like mimicking or sampling, but according to [1] and [2], imitation is discriminated from others by abstraction, conceptualization and symbolization. In fact, perfect imitation is accompanied by comprehension and generalization which are attained by abstraction. Hence, skills can be represented in a generalized symbolic level which is desired for high level cognitive tasks [3]. In addition, abstraction helps for efficient memory management, handling the huge real world search spaces [2], and quick knowledge transfer from an agent to another agent or from a situation to another situation [4].

Recently, symbolization and conceptualization has drawn attention in robot learning by imitation [2], [4]–[9]. In this respect, Hidden Markov Models (HMMs) have been extensively used for development of imitation models [2], [6], [8], [9]. It is because HMMs are powerful statistical tools for abstraction, generalization, recognition and generation of spatio-temporal signals. They can deal simultaneously with the statistical variations in the dynamics and the statistical variations in the observations. Consequently, HMMs can provide a unified mathematical formulation for learning from imitation. For example, Kulic et al. [8] developed an algorithm for incremental and autonomous learning, clustering, symbolization, recognition, organization, and regeneration of whole body motion patterns, using HMMs. However, the majority of previous works on conceptual imitation learning are dedicated to form concepts based on similarity in perceptual space, and there is not enough work to find abstract concepts which share functional properties. We think that although perceptual categorization is necessary to abstract demonstrations in imitation, however, there exists knowledge (e.g. functional meaning or effect of the action) which cannot be transferred merely by perceptual information.

The concepts which consider both perceptual and functional characteristics are called relational concepts. In fact, this paper aims to learn these kind of concepts through imitation. To our knowledge, there are only two works for conceptual imitation learning based on relational concepts. The first one was proposed by Mobahi et al. [5], [7] who introduced a bio-inspired model to acquire abstract relational concepts from teacher demonstrations, using reinforcement learning. However, unlike our procedure which is suitable for sequence of observations (e.g., human motion), their proposed algorithm is only applicable for concept acquisition from single observations. The second work which is the closest to ours has been introduced in [9]. This model identifies functional similarity between demonstrations through interaction with the teacher which is somehow tedious. But in our proposed model, functional similarity is specified by classifying the effects of actions which is more implicit and user-friendly. For more detailed review of related works on imitation and conceptualization, the interested reader is referred to [8] and [9].

Hence, the aim of this paper is to propose an incremental and gradual learning algorithm for autonomous acquisition, generalization, recognition and regeneration of relational concepts through perception of spatio-temporal demonstrations based on their functional effects. Perceptual abstraction of demonstrations is fulfilled stochastically by HMMs. However, an abstract (relational) concept is obtained as a collection of HMMs which might represent different perceptual properties but show the same functionalities. Functional similarities between different demonstrations are identified by recognizing effect of the executed actions. In the proposed algorithm, the concepts and proto-symbols emerge automatically without explicit human intervention. Also, the algorithm is invariant to the order of incoming demonstrations and acquires the concepts in parallel.

This paper is organized as follows. In section II, some basics and theories about concepts are reviewed. In addition, conceptual imitation is elaborated, and an approach is introduced to teach a concept oriented agent. Section III describes the proposed algorithm for learning and recall phases. In section IV, an experimental scenario is introduced to evaluate performance of the model. Also, results of the experiments, including abstraction, recognition, and generation of concepts are presented in this section. Finally, conclusions are drawn in section V.

II. Conceptual Imitation

A. Concepts

According to representational theory of mind, concept is a mental representation of world in agent's mind. It can be an idea, object, or event generally defined as a unit of meaning or knowledge [10]. This unit is constructed based on other units which describe some characteristics about the concept. In fact, these physical and/or functional characteristics make principles to categorize perceptions from world into concepts. For concept acquisition in natural environments, three points are desired [11]. First, concepts should be learned gradually as experience of the agent is increasing during the lifetime. Second, the concepts should be learned in parallel to cope with the diversity in type and order of incoming knowledge. Finally, like any learning procedure, it is very favorable to learn fast.

Concepts are categorized into three levels of abstraction, namely, perceptual, relational, and associative [10]. Perceptual concepts are formed based on similarity of instances in perceptual space. Relational concepts are formed not only by perceptual similarity but also by external information showing functional properties. However, in associative concepts, physical similarity is not important, but shared functional characteristics of the concepts are influential. An illustration of three types of concepts is provided in Fig. 1.

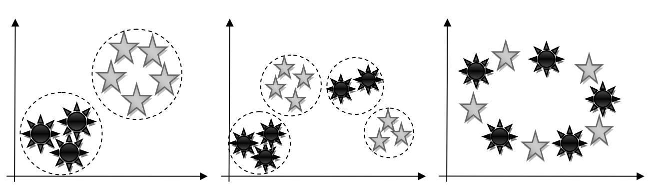

Fig. 1. Three types of concepts (from left to right): Perceptual, Relational, and Associative [9].

An important problem with a concept is how to represent it. Three theories are proposed in [12] to represent the concepts: exemplar, prototype, and rule theories. In exemplar theory, all instances of a concept are memorized. In prototype theory, a summary of instances are derived to represent various instances of a concept. This theory is more abstract and efficient to come up with limitations in memory. Finally, rule theory uses a match/mismatch process or boundary specification to represent concepts.

B. Problem Description

In this work, we want to devise an algorithm for autonomous learning and acquisition of relational concepts from imitation. In this way, demonstrated spatio-temporal behaviors are abstracted based on similarity in both perceptual and functional space. To this end, we favor to represent concepts by prototypes. Actually, the ideal situation is when we have the least number but the most general prototypes to understand a concept. Consequently, in the face of new demonstrations, the previously learned concepts can be recognized using generated proto-symbols, and there is no need of learning the behavior (motor commands to perform the behavior) from scratch. Also, behaviors which are associated with the same concept can be used alternatively in place of each other according to robot's comfort or affordance.

The real world is full of spatio-tempral experiences with relational concepts. For example, there are perceptually different actions which have the same functional effect in the environment (e.g., there are different body gestures that make people laugh). In our everyday life, we are facing with instances of these concepts permanently. A robot which is an inhabitant of the human environment will also faces similar experiences during colocation and interaction with the human over its entire lifespan. Hence, there should be an incremental and gradual mechanism to learn and acquire these concepts.

C. How to Teach Relational Concepts?

As described in section II-A, relational concepts cannot form merely from perceptual observations, and external information should be also provided to understand their functional characteristics. This information can unify perceptually scattered prototypes which represent the same concept. We think that the external information can be obtained through interaction with the teacher or investigation of the effects of demonstrated actions in the environment. For the former, Hajimirsadeghi et al. [9] proposed an interactive learning algorithm inspired from same/different judgement to identify functional similarity between demonstrations. However, in this paper we are focused on the effects of actions. In fact, if the agent can recognize the effect of demonstrated actions in the environment, it will be no need to interact with the teacher. In this case, the agent classify the demonstrations with similar effects in the same concepts. More details are explained in the next section.

III. The Proposed Algorithm

In this algorithm, HMMs are used for abstraction and symbolization of spatio-temporal perceptions. As a result, relational concepts are represented by HMM exemplars and prototypes which might encode different perceptual information but demonstrate the same functional properties. People unfamiliar with HMM should refer to [13]. Also, to find the algorithms for motion generation through HMM, one might see [2], [8].

A. Learning Phase

The learning algorithm is an iterative procedure where a cycle is repeated whenever a new demonstration is perceived. To ease explaining the learning algorithm, assume we are at the middle of execution where some concepts have been formed, and some prototypes and exemplars have been stored in the agent's memory. In our algorithm, an exemplar is an HMM made up of only one demonstration. However, prototypes are HMMs formed by unifying perceptually similar exemplars in the memory. Accordingly, we store the exemplars and prototypes in two different sets, namely Working Memory (WM) and Long-Term Memory (LTM), respectively. Finally, each concept is defined as a collection of prototypes and exemplars, and all the concepts together make the set of concepts Q:

$$WM = \bigcup \lambda_m; \lambda_m \text{ is an HMM exemplar,} \tag{1}$$
$$LTM = \bigcup \lambda_m; \lambda_m \text{ is an HMM prototype,} \tag{2}$$
$$Q = \bigcup q_k; q_k \text{ is a concept formed by a number of HMM prototypes and exemplars.} \tag{3}$$

However, to have better understanding of the algorithm, the concepts have been embodied into symbolic units in Fig. 2, and the exemplars and prototypes have been connected to them. In fact, in the proposed algorithm, the exemplars and prototypes membership in the concepts are described by two functions c^W and c^L which associate exemplar and prototype indexes with concept indexes, respectively:

$$c^W : \mathbf{N}_{|WM|} \to \mathbf{N}_{|Q|}, \tag{4}$$
$$c^L : \mathbf{N}_{|LTM|} \to \mathbf{N}_{|Q|}, \tag{5}$$
$$\mathbf{N}_{\mathcal{I}} := \{1, 2, \cdots, \mathcal{I}\}; \mathcal{I} \in \mathbb{N}. \tag{6}$$

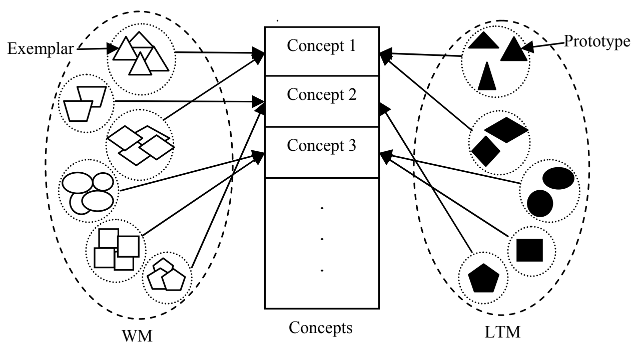

Fig. 2. Associative memory of exemplars, prototypes, and concepts.

Considering above definitions and assumptions, a schematic illustration of the algorithm is provided in Fig. 3. Detailed explanation of this algorithm is presented in the following paragraphs.

Assume that a novel demonstration is perceived by the robot. First, the effect of the demonstrated action is recognized. Then, it is investigated if there is any equivalent sensory-motor concept for the recognized effect in the memories. If there is no equivalent concept, a new concept should be generated. For this purpose, the new perceived demonstration is encoded into an HMM, is stored as an exemplar in the WM and make the new concept.

On the other hand, if there is an equivalent concept (e.g. q_k) for the recognized effect, this concept should be modified according to the new perception. To this end, first it is probed if there is any prototype in the LTM which can absorb the perceived demonstration. Hence, the likelihood of the perception sequence ($\mathbf{x} = x_1 x_2 \cdots x_T$) is computed against all the HMM prototypes of this concept, using forward algorithm. Next, the most probable prototype is selected according to (7):

$$i = \underset{m, \lambda_m \in LTM, c^L_m = k}{\arg\max} P(\mathbf{x}|\lambda_m). \tag{7}$$

Now, it is proposed that if the likelihood of this prototype is high enough, the perception sequence can be absorbed by the prototype. To evaluate whether the likelihood is high enough or not, the following criteria is used. If the log likelihood of the new perception given the winning HMM prototype is greater than the minimum log likelihood of that HMM's contents (i.e., the perceptions previously encoded in that HMM), the likelihood of this prototype is said to be high enough. We call the aforementioned minimum log likelihood value ll_min which is adjusted whenever a new HMM prototype is generated or modified. Hence, the following two cases are considered.

- *The Log Likelihood of the Winning Prototype is greater than ll_min :*
  In this case, the only thing to do is to strengthen (i.e. retrain) that HMM prototype by the new spatio-temporal perception. To this end, a modified form of re-estimation formulas suited for multiple observation sequences can be used [13]. The algorithm works by over-weighting the HMM prototype in order to consider the fact that it is built from multiple sequences.
- *The Log Likelihood of the Winning Prototype is less than ll_min :*
  This is the case when the log likelihood of the winning HMM prototype is not high enough to absorb the new perception. In this situation, the perception sequence is encoded as a new HMM exemplar, stored in the WM, and linked to the equivalent concept (i.e., c^W is modified). The reason to separate this case from the previous one is that there might have formed no prototype in the memory for the new perception yet, and so the most probable prototype is not really a representative for that perception sequence.
  Also note that if the equivalent concept has no prototype in the LTM, again the routine explained in this case (i.e., making a new exemplar) is carried out.

Following the procedure explained so far, the WM is overpopulated with exemplars after a short time. So, we must have an abstraction and consolidation mechanism to merge HMM exemplars and make HMM prototypes which are stored in the LTM. For this purpose, whenever an exemplar is stored in the WM of a concept and the number of exemplars associated with that concept exceeds a threshold number (Num_{th}), then a clustering process gets started on both HMM exemplars and prototypes of that concept. In this work, we use a mechanism similar to the algorithm proposed in [8] to cluster HMMs based on the pseudo-distance:

$$D\left(\lambda_1,\lambda_2\right)=\frac{1}{T}\left[\log P\left(\mathbf{x}^1|\lambda_1\right)-\log P\left(\mathbf{x}^1|\lambda_2\right)\right], \quad (8)$$

where, λ_1 and λ_2 are two HMM models, $\mathbf{x}^1$ is a perception sequence generated by λ_1, and T is the length of $\mathbf{x}^1$. Finally a symmetric distance is defined as:

$$D_s=\frac{D\left(\lambda_1,\lambda_2\right)+D\left(\lambda_2,\lambda_1\right)}{2}. \quad (9)$$

Now that the distances between HMMs are specified, an agglomerative algorithm which performs a complete link hierarchical clustering is used to construct new prototypes. Final clusters are selected based on two criteria, i.e., surpassing the minimum number of elements and falling behind the maximum distance measure. Maximum distance measure is defined according to the mean (μ_D) and standard deviation (σ_D) of the distances between all the HMMs in the concept:

$$D_{cutoff}=\mu_D-K_{cutoff}\cdot\sigma_D. \quad (10)$$

After this operation, if new clusters are produced, new HMM prototypes are trained with their associated elements in the clusters, using Baum-Welch algorithm or modified re-estimation formulas explained before. These consolidated prototypes are stored in the LTM.

In the schematic illustration of the algorithm, there are also some other functions, namely **New_W** and **New_C** to make new exemplars and concepts, respectively:

$$\mathbf{New\_W}:\Omega\left(\mathbf{x}^{dim}\right)\rightarrow WM, \quad (11)$$
$$\mathbf{New\_C}:=WM\rightarrow Q, \quad (12)$$

where, $\Omega\left(\mathbf{x}^{dim}\right)$ denotes the set of all finite sequences (or spatio-temporal signals) with dimension dim. Hence, in **New_W** function, a new HMM is trained with the perceived dim-dimensional sequence and stored in the WM.

B. Recall Phase

After learning process is accomplished, the robot will be capable of recognizing and predicting concept (i.e. effect) of novel demonstrations without the external information. For this purpose, HMM prototypes in the LTM are used. First, the likelihood of the perceived motion sequence against HMM prototypes is obtained through forward algorithm. Next, the perceived motion is recognized as one of the learned concepts by selecting the concept associated with the most probable prototype. In addition to recognition and prediction of concepts, the robot can use the acquired knowledge to reproduce the concepts. To this end, the robot employs the HMM prototypes of each concept to generate generalized motion patterns which can be transformed into motor signals, using the robot's inverse models.

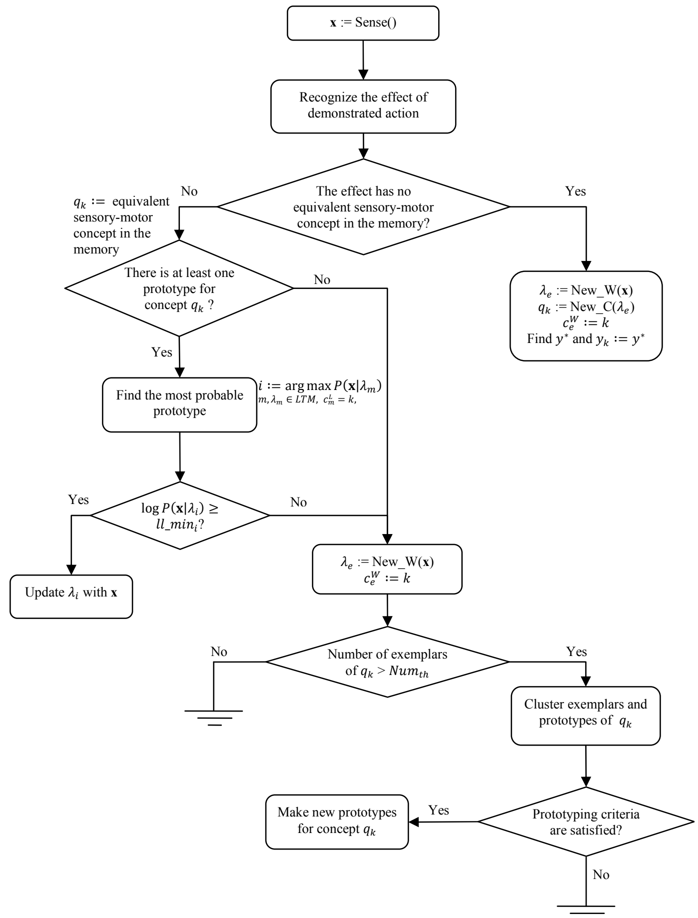

Fig. 3. Schematic Diagram of the concept learning algorithm.

IV. Experimental Studies

To evaluate the proposed algorithm for imitation learning of relational concepts, we set up an experiment. There are three agents in this experiment: A robot, a teacher, and a third human agent. The robot is a humanoid robot, namely the Nao academic robot. Demonstrations are provided for the robot by kinesthetic teaching. It means that the teacher grabs the robot's hand and performs an action, e.g. strikes at the third agent. Next, the third agent shows an emotional response to the the teacher's action, e.g. gets angry. The robot perceives its arm joint variables during performing the action and also recognize the emotional response of the third agent after action execution. So, the robot can detect functional similarities between different demonstrations based on the sameness of their emotional effects. For example, whether the teacher strikes the third agent from right or left, emotional response of the third agent will be the angry facial expression. All the concepts accompanied by their actions and emotional responses are summarized in Table I. In all the concepts, feedback of the third agent is identified by his facial expression except for the Love concept. For this concept, the response is to stimulate the tactile sensor on the Nao's head by caressing.

For facial expression detection in this experiment, the simple algorithm of "Eigenfaces" [14] is employed after cropping the face image, using the algorithm in [15].

A. Results

We employed our proposed algorithm to learn the concept of demonstrated actions. Total number of demonstrations in this experiment was 135, i.e., 15 demonstrations for each action. Each demonstration was perceived by the robot's kinesthetic sensory system during an action execution. As a result, perceptions were 4D signals made up of roll and pitch angles of the elbow and shoulder joints in the robot's right arm. Finally, the perceived signals incrementally entered to the learning algorithm. In the concept learning algorithm, we chose $K_{cutoff} = 0.5$, $Num_{th} = 3$, and the number of states for HMMs was set to 10. Minimum number of elements to form a new cluster (HMM prototype) was set based on the following rule. There should be at least one prototype and one exemplar or three exemplars in a candidate cluster to make a new prototype. We used k-fold cross validation with $k = 5$ to evaluate the performance of our algorithm for abstraction and recognition of the concepts. So, the experiment was repeated five times with different combinations of demonstrations for training and test.

Results of the experiments are summarized as follows. To show the progress of recognition accuracy during learning, a scoring mechanism is used. It means that whenever a demonstration is perceived, first it is classified by the previously learned concepts before entering to the learning algorithm. If the perception sequence is classified correctly, a positive score (+1) is issued, else a negative score (-1) is recorded. The scores (average of five experiments) recorded over the learning process on the training data is illustrated in Fig. 4. Note that due to the discrete nature of the scores, the result in the figure is smoothed with a window length of 10 to clearly reflect the expected behavior. It can be observed that the robot recognizes concept of all the new perception sequences correctly after training by a number of demonstrations.

Number of HMM prototypes produced at the end of the learning process of each experiment is listed in Table II. In most cases, the algorithm finds the same number of HMM prototypes as the number of actions which represent perceptual variants of a concept. In sum, however, there are sometimes one or two prototypes more than what is expected. For example, in the fifth experiment, two prototypes emerge for the concept of Unhappiness, but there is one action associated to this concept according to Table I. This outcome is because of the fact that some of the teacher's demonstrations are more similar to each other than the others, and so they are grouped into the same cluster during the learning process. We also illustrate the proto-symbol space of HMMs [6] for the fifth experiment in Fig. 5. This space is constructed based on the

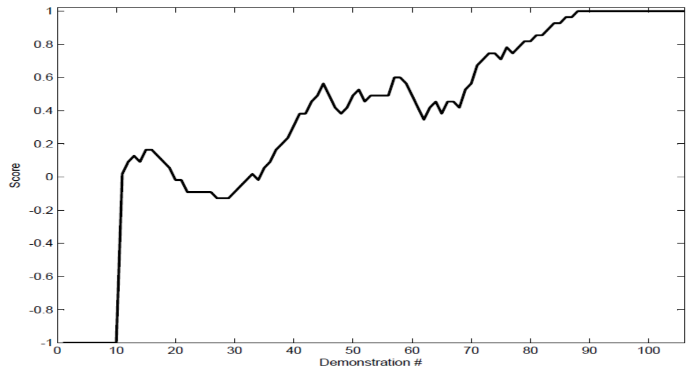

Fig. 4. Recorded scores over demonstrations.

distances between all pairs of HMM prototypes, using classical multidimensional scaling method [16]. In Fig. 5, the first three principal coordinates of multidimensional scaling are used to visualize dissimilarity of HMMs in the proto-symbol space.

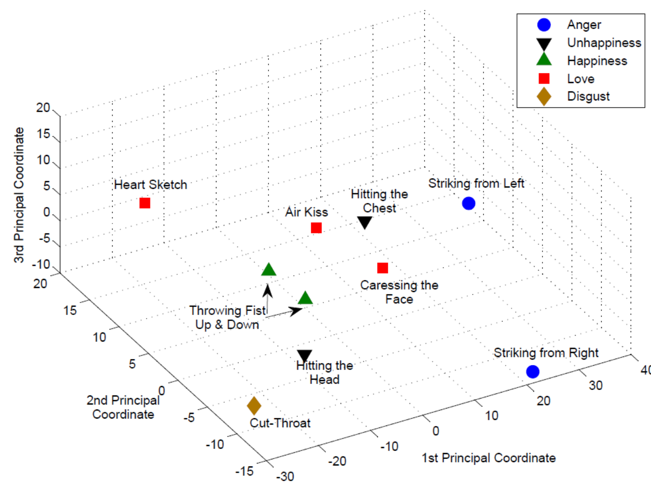

Fig. 5. Proto-symbol space of HMMs in the LTM for the fifth experiment.

To summarize performance of our proposed method, recognition accuracy of the algorithm for classifying the concepts in the test data is provided in Table III. This table also shows some statistics about the number of generated exemplars and prototypes in the WM and the LTM. Finally, samples of reproduced actions by the Nao academic robot accompanied by the third agent's emotional responses are demonstrated in Fig. 6.

V. Conclusion

In this study, we introduced a model for conceptual imitation. The main contribution was to devise an incremental and gradual learning algorithm for autonomous learning and acquisition of relational concepts from spatio-temporal demonstrations based on their functional characteristics. Functional similarity between demonstrations was identified by recognizing the effects of executed actions. HMMs were used to abstract spatio-temporal demonstrations into stochastic

TABLE I
ALL THE CONCEPTS ACCOMPANIED BY THEIR ACTIONS AND RESPONSES

#	Concept	The Third Agent's Response	Action 1	Action 2	Action 3
1	Anger	Angry Face	Striking from Right	Strike from Left	-
2	Unhappiness	Unhappy Face	Hitting the Head	Hitting the Chest	-
3	Happiness	Happy Face	Throwing Fist Up and Down	-	-
4	Love	Caressing the Robot's Tactile Sensor	Sketching Heart Sign	Air Kiss	Caressing the Face
5	Disgust	Disgusted Face	Cut-Through Gesture	-	-

TABLE II
NUMBER OF HMM PROTOTYPES GENERATED FOR EACH CONCEPT

Experiment no.	Anger	Unhappiness	Happiness	Love	Disgust	Sum
1	2	2	1	3	2	10
2	2	2	1	3	1	9
3	2	2	2	3	2	11
4	2	2	2	3	2	11
5	2	2	2	3	1	10

TABLE III
STATISTICAL INFORMATION FOR THE EXPERIMENTS WITH 5-FOLD CROSS VALIDATION

Accuracy		Size of WM		Size of LTM	
Mean %	Std %	Mean	Std	Mean	Std
100.00	0.00	18.00	5.92	10.20	0.84

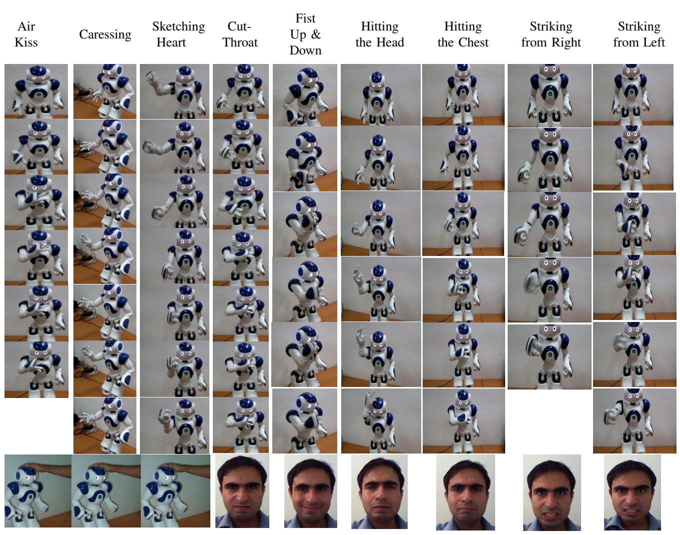

Fig. 6. Samples of reproduced actions by the Nao academic robot.

perceptual prototypes and exemplars. Consequently, relational concepts formed as a collection of irregularly scattered HMMs unified based on their functional effects. This abstraction leads to efficient memory management, generalization of acquired information, ease of knowledge transfer, and flexibility of choice between different alternatives. Finally, we evaluated the algorithm in an experimental scenario, namely conceptual hand gesture imitation learning by identifying their emotional effects. The experiments conducted on the Nao academic robot showed the successful results of our model for learning and acquisition of all the concepts and recognition and regeneration of their associated actions. As a results, the robot transforms to an effective agent which is capable of predicting effects (concepts) of novel demonstrations and also realizing theses effects by execution of appropriate actions.

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