Cooperative Coevolutionary Invasive Weed Optimization and its Application to Nash Equilibrium Search in Electricity Markets

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Abstract—**This paper presents a coevolutionary algorithm named cooperative coevolutionary invasive weed optimization (CCIWO) and investigates its performance for global optimization of functions with numerous local optima and also Nash equilibrium (NE) search for games. Ability of CCIWO for function optimization is tested through a set of common benchmarks of stochastic optimization, and reported results are compared with two other coevolutionary algorithms. In advance, a three-bus transmission-constrained electricity market model is studied, and CCIWO is employed to find NE for this complex system. Experimental results show efficiency of the proposed method to have more accurate solutions.**

Keywords-invasive weed optimization; coevolutionary agorithms; Nash equilibrium; biomimicry;

I. INTRODUCTION

Coevolutionary algorithm (CEA) is defined as "an evolutionary algorithm that employs a subjective internal measure for fitness assessment [1]." The term *subjective internal measure* means that fitness for the individuals are measured based on their interaction with each other and fitness value influences their evolution in some way. In this paper we focus on multi-population models in which the fitness for individuals is measured by their interaction with individuals in other populations. CEAs are usually discussed within two models: 1) cooperative and 2) competitive. In cooperative approach, a number of subpopulations representing pieces of the original problem evolve with an interactive scheme to solve the whole problem [1]. On the other hand, in competitive approach, value of the individuals is evaluated based on direct competition among individuals selected from evolving populations.

In recent years, with increasing growth of bio-inspired computing in a variety of applications, a considerable amount of attention has dedicated to evolutionary programming and computational intelligence for learning and simulation of games in electricity markets. Coevolutionary programming is the most popular technique for this purpose when demand functions are nonlinear or transmission constraints are binding. In [2], a hybrid coevolutionary algorithm is applied to solve constrained-transmission electricity markets, and in [3], a GA-based coevolutionary algorithm is exploited to simulate a simple electricity pool.

In this paper we are going to use invasive weed optimization (IWO) as an evolutionary part of a CEA and propose a cooperative coevolutionary IWO. IWO is a novel ecologically inspired algorithm that mimics the process of weeds colonization and distribution. Despite its recent development, it has shown successful results in a number of practical applications like optimization and tuning of a robust controller [4], distributed identification and adaptive control of a surge tank [5], task assignment of multiple UAVs [6], etc.

Section II provides steps for algorithm design comprising quick review of IWO, introduction to cooperative CEAs, and discussion on selection of collaborators in the proposed algorithm. In section III, simulation results for optimization of some common benchmarks are presented and compared with two other coevolutionary algorithms. Section IV is dedicated to Nash equilibrium search for transmission-constrained electricity markets, and finally conclusions are drawn in section V.

II. ALGORITHM DESIGN

A. Invasive Weed Optimization

IWO was developed by Mehrabian and Lucas in 2006 [4]. IWO algorithm is a numerical stochastic search algorithm mimicking natural behavior of weeds in colonizing and finding suitable place for growth and reproduction. Some of distinctive properties of IWO in comparison with other EAs are the way of reproduction, spatial dispersal and competitive exclusion [4]. These properties have shown superiority of IWO algorithm for switching between exploration and exploitation in the previous works [4]–[6]. Hence, we are motivated to introduce cooperative coevolutionary invasive weed optimization (CCIWO), hoping IWO improves searching capability in CEAs, and on the other hand, coevolutionary framework prepares a suitable basin of parallel computation and simulation of multiagent systems (like markets) for IWO.

In IWO, the process begins with initializing a population. It means that a population of initial solutions is randomly generated over the problem space. Then each member of population produces seeds depending on its relative fitness in the population. Number of seeds for each member varies between S_{min}, for the worst member of population, and S_{max}, for the best member of population. Seeds are randomly scattered in solution space by normally distributed

random numbers with mean equal to zero. Standard deviation (SD) of normal distribution for each generation is determined by equation (1).

$$\sigma_{iter} = \frac{(iter_{max} - iter)^n}{(iter_{max})^n} (\sigma_{init} - \sigma_{final}) + \sigma_{final} \qquad (1)$$

$iter_{max}$ is the maximum number of iterations, σ_{iter} is the SD at the current iteration and n is the nonlinear modulation index. The produced seeds and their parents considered as the potential solutions for the next generation. Finally, after a number of iterations the population reaches its maximum and an elimination mechanism should be employed. For this purpose, the seeds and their parents ranked together and those with better fitness survive and become reproductive [4]. The pseudocode for IWO is presented in Fig. 1, and the set of parameters for IWO algorithm is provided in Table I.

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1. Genearte random population of N_0 solutions;
2. For $iter$=: 1 to the maximum number of generations
   a. Compute maximum and minimum fitness in the colony;
   b. For each individula $w \in W$
      i. Compute number of seeds for w according to its fitness;
      ii. Randomly distribute generated seeds over the search space with normal distribution around the parent plant w;
      iii. Add the generated seeds to the solution set, W;
   c. If $(|W| = N) > p_{max}$
      i. Sort the population W in descending order of their fitness;
      ii. Truncate population of weeds with smaller fitness until $N = P_{max}$;
3. Next $iter$;

Figure 1.   Psuedocode for IWO algorithm

TABLE I.   IWO PARAMETERS

Symbol	Definition
N_0	Number of initial population
$iter_{max}$	Maximum number of iterations
P_{max}	Maximum number of plants
S_{max}	Maximum number of seeds
S_{min}	Minimum number of seeds
n	Nonlinear modulation index
σ_{init}	Initial vale of standard deviation
σ_{final}	Final vale of standard deviation

B. Cooperative Coevolutionary algorithm

In cooperative CEA, each population represents piece of a larger problem, and the populations evolve their own pieces in interaction with each other to solve the larger problem. A general cooperative coevolutionary framework is explained by its pseudocode in Fig. 2 [1].

For evolutionary part, each individual is combined with its collaborators from other populations to form a complete solution and the objective function is evaluated. Terminating criterion can be satisfied by falling short of the acceptable tolerance for changes in solutions or exceeding the maximum number of iterations. In evolutionary process, any EA can be exploited. For example, [7] uses GA for evolutionary part of CEAs, and [8] embeds PSO in the proposed CEA. To put optimization algorithms in the framework described in Fig. 2, each dimension is represented by a population of solutions. The fitness of each solution is evaluated by selecting the collaborating dimensions from other populations.

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1. For population $p_S \in P$, all populations
   a. Initialize population p_S;
2. For population $p_S \in P$, all populations
   a. Evaluate population p_S with collaborators;
3. For $t := 0$ until a terminating criterion is met
   a. For population $p_S \in P$, all populations
      i. Evolutionary process to make next generation;
      ii. Evaluate next generation with collaborators;
4. Next t

Figure 2.   Pseudocode for cooperative CEA [6].

Selecting the collaborators is very important in coevolutionary programming to have the best performance and find the true solutions. In [1], a number of attributes for this purpose are named: sample size, selective pressure and credit assignment. Sample size determines the number of collaborators, while selective pressure is the bias imposed on selection procedure, and credit assignment deals with the fact how to assign one fitness value to each individual from the results of multiple objective function evaluation (i.e. from interaction with multiple collaborators). There are usually two cases considered for this purpose: 1) collaborators are selected at random and 2) the best solutions from the last evaluation are taken as the collaborators. The former was studied in [2] and [3], while the latter was applied in part of the proposed hybrid coevolutionary algorithm with GA and hill climbing in [2] for Nash Equilibrium search in games. We use the second case in our CCIWO algorithm which has been shown to be much superior within our experiments.

For parameters setting, following the guidelines presented in [4] and [5] and also our experimental studies, some suggestions can be offered. Firstly, the best and general value for n is 3. Also, it is suggested to set S_{min} to 0 or 1 and S_{max} to 3. σ_{init} and σ_{final} are fixed according to the problem range of solutions and complexity of the function.

III.   SIMULATION RESULTS OF CCIWO FOR FUNCTION OPTIMIZATION

In this section, five test functions commonly used as benchmarks in evolutionary computation literature are considered, and simulation results for minimization of them with CCIWO are compared with two other CEAs. These functions are described in Table II.

Cooperative coevolutionary GA (CCGA) and a competitive coevolutionary algorithm named loosely coupled GA (LCGA) introduced in [7] are two CEAs employed for comparison. The experiments are conducted with number of variables equal to 30 for all test functions. Each experiment is performed for 25 trials with 200 generations and the best 20 trials are averaged to represent the results (cf. Table III). It can be observed that CCIWO has better results than other

methods for optimization of these test functions. Parameters for this experiment are shown in Table IV.

TABLE II.   TEST FUNCTIONS

Name	Function	Initial range	Modality
Sphere	$\sum_{i=1}^{n}(x_i^2)$	[-100, 100]	unimodal
Rosenbrock	$\sum_{i=1}^{n}[100(x_{i+1} - x_i^2)^2 + (x_i - 1)^2]$	[-2.12, 2.12]	unimodal
Rastrigin	$\sum_{i=1}^{n}(x_i^2 - 10\cos(2\pi x_i) + 10)$	[-5.12, 5.12]	multimodal
Ackley	$-20e^{\left(-0.2\sqrt{\frac{1}{n}\sum_{i=1}^{n}x_i^2}\right)} - e^{(\frac{1}{n}\sum_{i=1}^{n}\cos 2\pi x_i)} + 20 + e$	[-32, 32]	multimodal
Griewank	$1 + \sum_{i=1}^{N}\frac{x_i^2}{4000} - \prod_{i=1}^{N}(\cos(x_i/\sqrt{i}))$	[-600, 600]	multimodal

TABLE III.   RESULTS FOR COMPARISON WITH CCGA AND LCGA

Function	Mean value			Eval. number[a]		
	CCGA	LCGA	CCIWO	CCGA	LCGA	CCIWO
Sphere	1e-08	1e-12	4e-13	600000	600000	326203
Rosenbrock	70	90	0.27	600000	600000	323635
Rastrigin	0.5	0.12	4e-10	600000	600000	324578
Ackley	0.8	8	3e-07	600000	600000	316616
Griewank	0.02	2	2e-12	600000	600000	323789

a. Number of local function evaluations for LCGA and complete function evaluations for others

TABLE IV.   CCIWO PARAMETERS FOR FUNCTION OPTIMIZATION

Symbol	Value	Symbol	Value
N_0 (subpopulation)	20	n	3
$iter_{max}$	200	S_{min}	1
P_{max} (subpopulation)	20	S_{max}	3

Symbol	Sphere	Rosenbrock	Rastrigin	Ackley	Griewank
σ_{init}	10	0.5	1	2	30
σ_{final}	1e-7	5e-5	5e-5	5e-6	5e-5

IV.   CCIWO FOR NE SEARCH IN ELECTRICITY MARKETS

A. Games and Nash Equilibrium

A general multi-player game consists of an index set $N = \{1, 2, 3, ..., N\}$ called player's set and an index set $K = \{1, 2, 3, ..., K\}$ as the stages of the game, showing the allowable number of moves for each player. In each stage, players take strategies from a set of strategy spaces $U = \{U_k^i\}$, and receive a payoff of $\pi_i(u^i, u^{-i})$, where $u^i \in U^i$ is the pure strategy for player i, given pure strategy set of others $u^{-i} = \{u^1, ..., u^{i-1}u^{i+1}, ..., u^N\} \in U^{-i}$. Pure strategy Nash Equilibrium (NE) is a point where no player can obtain a higher profit by unilateral movement. The satisfying NE condition for the combined strategy $\{u^{i*}, u^{-i*}\}$ is characterized in (2).

$$\forall i, \forall u^i \in U^i, \qquad \pi_i(u^{i*}, u^{-i*}) \geq \pi_i(u^i, u^{-i*}) \qquad (2)$$

As we will use the term local NE in this paper, here a definition of that from [2] is also provided.

$$\exists \varepsilon > 0 \text{ such that } \forall i, \forall u^i \in B^{i,\varepsilon}(u^{i*}),$$
$$\pi_i(u^{i*}, u^{-i*}) \geq \pi_i(u^i, u^{-i*}) \qquad (3)$$

where $B^{i,\varepsilon}(\hat{u}^i) = \{u^i \epsilon U^i \mid \|u^i - \hat{u}^i\| < \epsilon\}$

B. Transmission-Constrained Electricity Markets

Although transmission-constrained electricity markets with linear demand functions have linear demand curves, but the transmission constraints can cause individual profit functions to have local optima [2]. Actually, reaction curves in this model are discontinuous piecewise linear functions that might make local NE traps or even disrupt existence of pure strategy equilibrium for the game [2], [9]. In addition to the fact that transmission-constrained electricity market model is a good mathematical example with a complex game structure and local optima, it is an important model for market power analysis in the restructured electricity industry. Hence, transmission-constrained electricity market is a good example of complex game for our purpose of Soft Computing. Shortly, trading in electricity markets can be represented by the maximization of total welfare subject to the constraints on the system (4).

$$\max\left(\sum_j Benefit_j - \sum_i Cost_i \right)$$

$$\text{S. T. } \begin{cases} \text{Transmission thermal limits} \\ \text{Total supply = total demand} \\ \text{Kirchoff's laws} \end{cases} \qquad (4)$$

When transmission constraints are binding in the imperfectly competitive market, Cournot behavior will produce locational price differences similar to a competitive market with constraints present. This increases the difficulty of computing the profit maximizing condition of the strategic players. The profit maximizing function of each strategic player has an embedded transmission-constrained welfare maximization problem within its major problem. The generation and transmission line constraints are included in the welfare maximization subproblem. The profit function maximization of each utility is given in (5).

$$\max\left\{ P_i q_i - Cost_i \left| \begin{matrix} \max\sum_j Benefit_j \\ \text{Transmission Constraints} \end{matrix} \right. \right\} \qquad (5)$$

Locational prices (P_i), are determined by the Lagrange multipliers of the locational energy balance equality condition for Kirchoff's laws in the welfare maximization problem which is also the market-clearing problem, here.

C. Three-Bus Cournot Model with Linear Demand Function

This is a complex model of a three-bus transmission-constrained electricity market with a generator and a load at each bus and a pure NE at $q_1 = 1106$, $q_2 = 1046$, $q_3 = 995$ which is solved in [2] and [9] with hybrid coevolutionary programming and graphic representation, respectively. This three-bus network is depicted in Fig. 3.

Cournot game of this system is characterized in profit maximizing behavior of the players described in (6).

$$\pi_1(q_1, q_2, q_3) = \lambda_1^* q_1 - C_1(q_1)$$
$$\pi_2(q_1, q_2, q_3) = \lambda_2^* q_2 - C_2(q_2) \qquad (6)$$
$$\pi_3(q_1, q_2, q_3) = \lambda_3^* q_3 - C_3(q_3)$$

where locational marginal prices $\lambda_1^*, \lambda_2^*, \lambda_3^*$ are determined by the Lagrange multipliers of energy balance equality conditions (the first three constraints) in the market clearing problem (7).

$$\max_d \left(B_1(d_1) + B_1(d_1) + B_1(d_1) \right)$$

$$\begin{aligned} \text{S.T.} \quad & q_1 - d_1 = 2T_1 - T_3 \\ & q_2 - d_2 = -T_1 + 2T_3 \\ & q_3 - d_3 = -T_1 - T_3 \\ & |T_1| < T_1^{max} \end{aligned} \qquad (7)$$

To find NE for this game with the proposed coevolutionary algorithm, each agent is represented by a population of bidding quantities as the decision variables, and profit functions are evaluated in the coevolutionary process explained in section II until no agent can get more profit by changing its quantity without changes of the quantities for other producers. In our algorithm, this termination criterion is approximated by defining a tolerance of changes for the best complete solution (union of best quantities in all the subpopulations) for a predefined number of iterations, i.e., when there is no significant change in the best complete solution for a number of iterations, the algorithm stops.
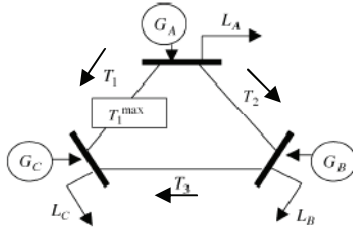
$$B_1(d_1) = -.0555d_1^2 + 108.4096d_1 \quad C_1(q_1) = 0.00786q_1^2 + 1.3606q_1$$
$$B_2(d_2) = -.0669d_2^2 + 103.8238d_2 \quad C_2(q_2) = .010526q_2^2 - 2.07807q_2$$
$$B_3(d_3) = -.0637d_3^2 + 105.6709d_3 \quad C_3(q_3) = .006478q_3^2 + 8.105354q_3$$

Figure 3.   Three-bus Cournot model [5].

Here, we use our proposed CCIWO to find NE in the case of $T_1^{max} = 100$. The coevolution process for CCIWO is shown in Fig. 4. It can be observed that our coevolutionary algorithm converges to the optimal solution after a limited number of iterations. Comparing with the previously proposed coevolutionary algorithms in [2], we can say that our algorithm is better than the simple coevolutionary genetic algorithm in finding the global NE and also outperforms the hybrid coevolutionary genetic algorithm in number of function evaluations and computational complexity.
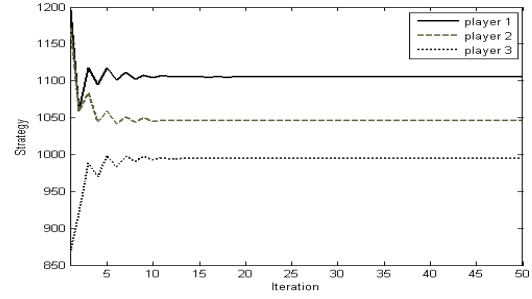

Figure 4.   CCIWO for the three-bus model with $T_1^{max} = 100$

V.   CONCLUSION

In this paper we proposed CCIWO, a coevolutionary algorithm derived from invasive weed optimization. Efficiency of CCIWO for global optimization was tested through challenging and popular test functions in stochastic optimization. Moreover, CCIWO was employed for Nash equilibrium search in a complex electricity market model. Results showed that CCIWO has a good performance for both purpose of global optimization and NE search within optimal precision of the solutions and lower computational load.

For future work, we are to study the proposed algorithm for NE search in mixed strategy games with numerous equilibria. In addition, analysis of transmission-constrained electricity markets with nonlinear demand functions is our current focus of research.

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